NAG Toolbox for MATLAB

g13cg

1 Purpose

For a bivariate time series, g13cg calculates the noise spectrum together with multiplying factors for the bounds and the impulse response function and its standard error, from the univariate and bivariate spectra.

2 Syntax

```
[er, erlw, erup, rf, rfse, ifail] = g13cg(xg, yg, xyrg, xyig, stats, l,
n, 'ng', ng)
```

3 Description

An estimate of the noise spectrum in the dependence of series y on series x at frequency ω is given by

$$f_{y|x}(\omega) = f_{yy}(\omega)(1 - W(\omega)),$$

where $W(\omega)$ is the squared coherency described in g13ce and $f_{yy}(\omega)$ is the univariate spectrum estimate for series y. Confidence limits on the true spectrum are obtained using multipliers as described for g13ca, but based on $(d - 2)$ degrees of freedom.

If the dependence of y_t on x_t can be assumed to be represented in the time domain by the one sided relationship

$$y_t = v_0 x_t + v_1 x_{t-1} + \cdots + n_t,$$

where the noise n_t is independent of x_t, then it is the spectrum of this noise which is estimated by $f_{y|x}(\omega)$.

Estimates of the impulse response function $v_0, v_1, v_2, \ldots$ may also be obtained as

$$v_k = \frac{1}{\pi}\int_0^{\pi} \mathrm{Re}\left(\frac{\exp(ik\omega) f_{xy}(\omega)}{f_{xx}(\omega)}\right),$$

where Re indicates the real part of the expression. For this purpose it is essential that the univariate spectrum for x, $f_{xx}(\omega)$, and the cross spectrum, $f_{xy}(\omega)$, be supplied to this function for a frequency range

$$\omega_l = \left[\frac{2\pi l}{L}\right], \qquad 0 \le l \le [L/2],$$

where [] denotes the integer part, the integral being approximated by a finite Fourier transform.

An approximate standard error is calculated for the estimates v_k. Significant values of v_k in the locations described as anticipatory responses in the parameter array **rf** indicate that feedback exists from y_t to x_t. This will bias the estimates of v_k in any causal dependence of y_t on $x_t, x_{t-1}, \ldots$.

4 References

Bloomfield P 1976 *Fourier Analysis of Time Series: An Introduction* Wiley

Jenkins G M and Watts D G 1968 *Spectral Analysis and its Applications* Holden–Day

5 Parameters

5.1 Compulsory Input Parameters

1: **xg(ng) – double array**

The **ng** univariate spectral estimates, $f_{xx}(\omega)$, for the x series.

2: **yg(ng) – double array**

The **ng** univariate spectral estimates, $f_{yy}(\omega)$, for the y series.

3: **xyrg(ng) – double array**

The real parts, $cf(\omega)$, of the **ng** bivariate spectral estimates for the x and y series. The x series leads the y series.

4: **xyig(ng) – double array**

The imaginary parts, $qf(\omega)$, of the **ng** bivariate spectral estimates for the x and y series. The x series leads the y series.

Note: the two univariate and bivariate spectra must each have been calculated using the same method of smoothing. For rectangular, Bartlett, Tukey or Parzen smoothing windows, the same cut-off point of lag window and the same frequency division of the spectral estimates must be used. For the trapezium frequency smoothing window, the frequency width and the shape of the window and the frequency division of the spectral estimates must be the same. The spectral estimates and statistics must also be unlogged.

5: **stats(4) – double array**

The four associated statistics for the univariate spectral estimates for the x and y series. **stats**(1) contains the degree of freedom, **stats**(2) and **stats**(3) contain the lower and upper bound multiplying factors respectively and **stats**(4) contains the bandwidth.

Constraints:

stats(1) ≥ 3.0;
$0.0 <$ **stats**(2) ≤ 1.0;
stats(3) ≥ 1.0.

6: **l – int32 scalar**

L, the frequency division of the spectral estimates as $\frac{2\pi}{L}$. It is also the order of the FFT used to calculate the impulse response function. **l** must relate to the parameter **ng** by the relationship.

Constraints:

ng $= [L/2] + 1$;
The largest prime factor of **l** must not exceed 19, and the total number of prime factors of **l**, counting repetitions, must not exceed 20. These two restrictions are imposed by c06eb which performs the FFT.

7: **n – int32 scalar**

The number of points in each of the time series x and y. **n** should have the same value as **nxy** in the call of g13cc or g13cd which calculated the smoothed sample cross spectrum. **n** is used in calculating the impulse response function standard error (**rfse**).

Constraint: **n** ≥ 1.

5.2 Optional Input Parameters

1: **ng – int32 scalar**

Default: The dimension of the arrays **xg**, **yg**, **xyrg**, **xyig**, **er**. (An error is raised if these dimensions are not equal.)

the number of spectral estimates in each of the arrays **xg**, **yg**, **xyrg**, **xyig**. It is also the number of noise spectral estimates.

Constraint: **ng** ≥ 1.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **er(ng) – double array**

The **ng** estimates of the noise spectrum, $\hat{f}_{y|x}(\omega)$ at each frequency.

2: **erlw – double scalar**

The noise spectrum lower limit multiplying factor.

3: **erup – double scalar**

The noise spectrum upper limit multiplying factor.

4: **rf(l) – double array**

The impulse response function. Causal responses are stored in ascending frequency in **rf**(1) to **rf**(**ng**) and anticipatory responses are stored in descending frequency in **rf**(**ng** $+ 1$) to **rf**(**l**).

5: **rfse – double scalar**

The impulse response function standard error.

6: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g13cg may return useful information for one or more of the following detected errors or warnings.

ifail $= 1$

On entry, **ng** < 1,
or **stats**$(1) < 3.0$,
or **stats**$(2) \leq 0.0$,
or **stats**$(2) > 1.0$,
or **stats**$(3) < 1.0$,
or **n** < 1.

ifail $= 2$

A bivariate spectral estimate is zero. For this frequency the noise spectrum is set to zero, and the contribution to the impulse response function and its standard error is set to zero.

ifail $= 3$

A univariate spectral estimate is negative. For this frequency the noise spectrum is set to zero, and the contributions to the impulse response function and its standard error are set to zero.

ifail $= 4$

A univariate spectral estimate is zero. For this frequency the noise spectrum is set to zero and the contributions to the impulse response function and its standard error are set to zero.

ifail = 5

A calculated value of the squared coherency exceeds 1.0. For this frequency the squared coherency is reset to 1.0 with the consequence that the noise spectrum is zero and the contribution to the impulse response function at this frequency is zero.

ifail = 6

On entry, $[\mathbf{l}/2] + 1 \neq \mathbf{ng}$,
or **l** has a prime factor exceeding 19,
or **l** has more than 20 prime factors, counting repetitions.

If more than one failure of types 2, 3, 4 and 5 occurs then the failure type which occurred at lowest frequency is returned in **ifail**. However the actions indicated above are also carried out for failures at higher frequencies.

7 Accuracy

The computation of the noise is stable and yields good accuracy. The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

8 Further Comments

The time taken by g13cg is approximately proportional to **ng**.

9 Example

```
xg = [2.0349;
     0.51554;
     0.0764;
     0.01068;
     0.0009300000000000001;
     0.001;
     0.00076;
     0.00037;
     0.00021];
yg = [21.97712;
     3.29761;
     0.28782;
     0.0248;
     0.00285;
     0.00203;
     0.00125;
     0.00107;
     0.00191];
xyrg = [-6.54995;
     0.34107;
     0.12335;
     -0.00514;
     -0.00033;
     -0.00039;
     -0.00026;
     0.00011;
     6.999999999999999e-05];
xyig = [0;
     -1.1903;
     0.04087;
     0.00842;
     0.00032;
     -1e-05;
     0.00018;
     -0.00016;
```

```
     0];
stats = [30;
     0.63858;
     1.7867;
     0.33288];
l = int32(16);
n = int32(296);
[er, erlw, erup, rf, rfse, ifail] = g13cg(xg, yg, xyrg, xyig, stats, l,
n)
```

```
er =
    0.8941
    0.3238
    0.0668
    0.0157
    0.0026
    0.0019
    0.0011
    0.0010
    0.0019
erlw =
    0.6298
erup =
    1.8291
rf =
   -0.0547
    0.0586
   -0.0322
   -0.6956
   -0.7181
   -0.8019
   -0.4303
   -0.2392
   -0.0766
    0.0657
   -0.1652
   -0.0439
   -0.0494
   -0.0384
    0.0838
   -0.0814
rfse =
    0.0863
ifail =
           0
```